

Paper - III

Real Analysis - The branch of Mathematics called real analysis deals with problems which are closely connected with the notion of 'limit' and some other notions, such as the operation of differentiation and integration, which are directly dependent on the concept of limit when all these operations are confined to the domain of real numbers.

Algebraic structure - A set is said to possess an algebraic structure if the two composition of Addition and Multiplication are defined in the set.

Order structure - Ordinarily the Order relation does not exist between the members of a general field. But as we are deal with the field of real numbers.

A field  $S$  is an ordered field if it satisfies the following properties -

(1) Law of Trichotomy - For any two elements  $a, b \in S$  one and only one of the following is true.  
 $a > b, a = b, a < b$

(2) Transitivity -  $\forall a, b, c \in S$   
 $a > b$  and  $b > c \Rightarrow a > c$

(3) Compatibility of Order Relation with addition  
Composition -  $\forall a, b, c \in S$   
 $a > b \Rightarrow a + c > b + c$

## Real Analysis

B.Sc - Part-II

<4> Compatibility of Order Relation with Multiplication

Composition:  $\forall a, b, c \in S$

$$a > b \Rightarrow ac > bc$$

and  $c > 0$

Eg. Set of Rational number i.e  $\mathbb{Q}$  and Set of Real number that is  $\mathbb{R}$  are Ordered fields, while the set  $\mathbb{N}$  of natural numbers and the set  $\mathbb{I}$  of integers are not fields.

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## Intervals Open and Closed

A subset  $A$  of  $\mathbb{R}$  is called an interval if  $A$  contains (i) at least two distinct elements  
(ii) Every element lies between any two members of  $A$ .

Open interval. - If  $a$  and  $b$  be two real numbers such that  $a < b$  then the set

$\{x: a < x < b\}$   
Consisting of all real numbers between  $a$  and  $b$  (excluding  $a$  and  $b$ ) is called an open interval and denoted by  $]a, b[$  or  $(a, b)$  i.e.  $(a, b)$

Closed interval The set

$\{x: a \leq x \leq b\}$   
Consisting of  $a, b$  and all real numbers lying between  $a$  and  $b$  is called a closed interval and is denoted by  $[a, b]$

Semi-closed or Semi-open intervals

$$]a, b] = \{x: a < x \leq b\}$$

$$[a, b[ = \{x: a \leq x < b\}$$

The intervals are semi-closed or semi-open. The former is open at  $a$  and closed at  $b$ . While the later is closed at  $a$  and open at  $b$ .

## Bounded and Unbounded Sets: Supremum, Infimum

A subset  $S$  of real numbers is said to be bounded above if  $\exists$  a real number  $K$  such that every member of  $S$  is less than or equal to  $K$  i.e.

$$x \leq K \quad \forall x \in S$$

The number  $K$  is called an upper bound of  $S$ . If no such number  $K$  exists, the set is said to be unbounded above or not bounded above.